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Learning Curves: An Alternative Analysis

Sharif F. Harris

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LEARNING CURVES: AN ALTERNATIVE ANALYSIS

THESIS

Sharif F. Harris, First Lieutenant, USAF
AFIT-ENV-MS-17-M-193

**DEPARTMENT OF THE AIR FORCE
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LEARNING CURVES: AN ALTERNATIVE ANALYSIS

THESIS

Presented to the Faculty

Department of Systems Engineering and Management

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Air Education and Training Command

In Partial Fulfillment of the Requirements for the

Degree of Master of Science in Cost Analysis

Sharif F. Harris, BS

First Lieutenant, USAF

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LEARNING CURVES: AN ALTERNATIVE ANALYSIS

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Abstract

Learning curves are used to describe and estimate the cost performance of a serial production process. There are numerous different models and methods, however, it is not definitively known which is preferred. The research objective is to performance compare the more common learning curve models. The research goal is improved understanding of the systemic cost drivers of a production process, their relationship to cost, and present modeling methods. The research method is qualitative analysis combined with statistical regression modeling. The research identified that preference for one function or another depended upon the shape of the data and how well a model formulation could be made to fit that shape. This depended upon the model's basic shape and the available parameters to alter its appearance. The typical learning curve model assumes that cost is a function of time but commonly omits factors such as production process resources changes (capital and labor) and its effect on cost. A learning curve model that includes the effects of resource changes would likely provide higher estimative utility given that it establishes a systemic relationship to the underlying production process. Additional research and data is required to further develop this understanding.

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LEARNING CURVES: AN ALTERNATIVE ANALYSIS

I. Introduction

On Friday 3 February 2017, the United States Department of Defense (DoD) and Lockheed Martin Corporation signed an \$8.2 billion contract for the next 90 F-35 fighter jets. This is the first time that the planes will have a sub \$100 million-unit cost and comes after President Donald Trump complained in December that program costs were “out of control” (Kaskovich, 2017). Lockheed Martin indicated that they planned on hiring 1,800 additional workers and that a cost reductive \$170 million capital investment program had concluded at year end 2016. Lockheed CEO, Marillyn Hewson stated that per unit prices have decreased 60% since the program’s initiation and that “this demonstrates a learning curve as efficient as any achieved on any modern tactical aircraft” (Weisgerber, 2017).

In this particular instance the phrase *learning curve* refers to the often observed decrease in unit cost of a serially produced item as additional units of that item are made. This is particularly prevalent during the earliest segment of production for a new product, which was the case for the F-35 program. Typically, this effect is attributed to increased familiarity with the product and its production method. However, management decisions to alter the production process through capital investment and increased employment has often been an underappreciated source of product cost reduction. Understandably, much attention is paid to learning curves, as they are used to describe and estimate the unit cost

of a production process. This research seeks to further develop the understanding and use of learning curves within the DoD cost estimating community.

Background

The principal objective of individuals and organizations is to maximize the value of their resources through effective management. Generally, businesses seek to maximize profitability and the productivity of employees and fixed investment. Governments desire the maximum public good of tax expenditures and investments. Managers depend heavily upon budgets and plans to assist resource management and decision making. Budgets and strategic plans are programs for future action and are estimate driven. The utility of either depends upon the accuracy and reliability of those estimates and assumptions from which they are formulated. Consequently, the success of a manager is particularly reliant on the overall quality of those estimates provided. Those past and presently ongoing Department of Defense (DoD) major weapons procurement programs exemplify this relationship.

Historically, the DoD has regularly underestimated development and procurement costs of major weapons systems (Arena and others, 2006:22). A 2006 RAND Corporation review of 68 Major Defense Acquisition Programs (MDAP) spanning from 1968 to 2003 identified that average quantity adjusted cost growth from milestone B estimate to project completion was 46% and was 16% from milestone C estimate to completion (Arena and others, 2006:21-22). A MDAP is a research and development effort expected to require in excess of \$480 million or a procurement exceeding \$2.79 billion in fiscal year (FY) 2014 constant dollars (DoD, 2015:44). The current annual Government Accountability Office

(GAO) DoD procurement assessment released 31 March 2016 indicates 79 active MDAP with a total value of 1.44 trillion in FY16 constant dollars (GAO, 2016:8). Milestone B is the point in the procurement process which research and development is mostly complete and a functioning prototype has been prepared. From here the product design is finalized, production methods chosen, and production begins. Milestone C is the point at which the design and production method are finalized. From this point mass production begins and the item is operationally fielded (DoD, 2005:16-30).

The presently ongoing F-35 Joint Strike Fighter (JSF) procurement program is a noteworthy example of cost underestimation. A 14 June 2012 (GAO) report indicates that in 2012 the total program cost was estimated at \$161 million per aircraft compared to the 2001 program initial baseline estimate of \$81 million per aircraft (GAO, 2012:5). This is an approximate 99% increase in an eleven-year span and additional increases have since occurred. Cost estimates that have a 46% post milestone B average increase for MDAPs, and the near doubling of estimated cost in the case of the F-35, are of limited utility to a decision maker.

The specific reasons for cost estimate deviation relative to the actual expenditures are numerous and system peculiar. Frequently, the cost deviation observed is attributable to influences well beyond the reasonable control and foresight of a cost estimator. A 2008 RAND Corporation study of 35 MDAP indicates that cost estimation error accounted for approximately 16.83% of the cost growth observed in those programs (Bolten and others, 2008:27). A similar 2004 study by David McNicol of 138 weapon systems procurements notes cost deviation as a result of mistakes (unrealistic estimates or poor management) in 70% of the systems reviewed. The average estimative deviation was -20% to 30% (Arena

and others, 2006:8). McNicol suggests that services showed a tendency toward optimistic estimates (Arena and others, 2006:15). Nevertheless, estimative errors have been shown to be at least partially responsible for total cost deviation, and improving the estimating methods and techniques most frequently used by the DoD is an important component of the effort to enhance estimate accuracy. The primary research focus are the more common serial production process cost estimating techniques used within the DoD. Typically, the final output of a weapon system procurement program, be it aircraft, vehicles, or ships, are manufactured through a serial production process. During production, unit costs are frequently observed to decline with incremental production. The general explanation for this phenomenon is that increased familiarity with production tasks enables the reduction of the time and cost to produce additional units (AFCAH, 2007:4). Given sufficient time, a production process stabilizes and individual unit costs generally remain constant going forward. This phenomenon is modeled into cost estimates and is usually referred to as the learning curve.

Problem and Hypothesis

Learning curves are developed using a statistical modeling technique called least squares regression. The objective is to fit a mathematical function to a data set in question by manipulating the function's controlling constants. The specific task is to minimize the total error (sum of squares of differences between mathematical functional output and the corresponding data points) in the model (McClave and others, 2014:606). The Coefficient of determination (R^2) is a measure of a model's explanatory power and fit quality relative

to the data set (McClave and others, 2014:634-636). Generally, data is sourced from the production process being analyzed or an analogous program. The model is then used to describe the data as well as estimate future expected unit costs for the production process. The power function is the mathematical function most regularly used by cost estimators to model the anticipated effects of learning in a production process. A power function is a mathematical function of the form shown below in Equation 1.

$$y = x^a \tag{1}$$

Where

y = function output
x = independent variable
a = exponential constant

Power function based learning models assign a negative value to the constant *a* which produces a convex curve shown below in Figure 1.

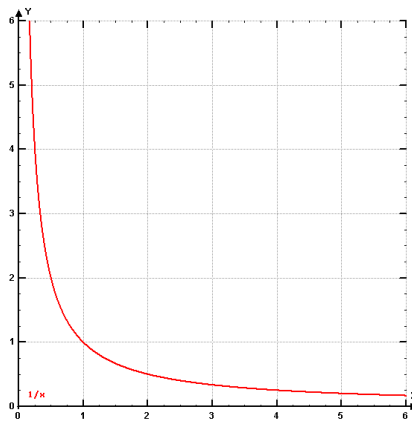


Figure 1. Power Function (Basic Form Example)

The shortfall of the power function, however, is that it does not exhibit behavior which could reasonably be interpreted as a long-term steady state. As the independent variable approaches positive infinity, the function output nears zero but does not visually stabilize

horizontally. However, both logical intuition and empirical observation have shown that a production process, with a constant set of capital and labor, will stabilize given sufficient time. The extent of improvements is identified over the lifetime of a specific production process are limited. Furthermore, the cost of additional or continual production process change would eventually exceeded the benefits, once what is reasonably construed as the most efficient arrangement is identified. Given that the modeled phenomenon eventually stabilizes with time whereas the power function does not, a divergence will emerge which widens with time making any estimate based thereupon increasingly inaccurate with each successive iteration.

A possible alternative might be a mathematical function that initially decreases with increasing independent values then stabilizes horizontally as the independent values approach infinity. A function of this nature would better approximate the typical expected learning behavior. A sigmoid function, or an s-curve, exhibits this behavior. A sigmoid function is one of the basic form shown in Equation 2.

$$y = \frac{1}{1 + e^x} \quad (2)$$

Where

y = function output
e = natural logarithm (constant)
x = independent variable

A graph of the curve produced by this function is shown below in Figure 2.

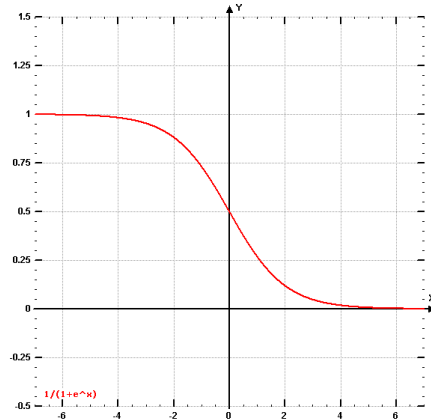


Figure 2. Sigmoid Function (Basic Form Example)

A sigmoid function unlike a power function begins and ends in what could be reasonably interpreted as a horizontal steady state phase. Additionally, a sigmoid function provides a practitioner increased control over the behavior of its shape, and potentially offers higher precision when modeling the learning effect compared to a power function. The increased control could possibly raise implementation difficulty compared to a power function, but the added value of comparative accuracy improvement will likely compensate. A sigmoid function is presumably preferred to a power function for modeling the cost behavior of a serial production process and is expected to have a higher coefficient of determination (R^2) value than that of a power function when modeled to the cost data for such a process.

Problem Statement and Research Questions

The DoD has historically underestimated the development and production costs of MDAPs. Approximately 16.83% of the historical error was attributed to cost estimates by the previously mentioned RAND study. This study, however, did not specifically mention any particular estimating methodology or technique most responsible for estimative error

making it difficult to develop generalizations or targeted assessments of specific methods. Learning curves are a common component of cost estimates, and are often applied to the production component of the acquisition process. Improving the estimating and modeling methodologies thereof would likely be beneficial to overall cost estimate accuracy. Minor variations to the learning curve modeled estimate can have significant ramifications on total estimated procurement cost (AFCAH, 2007:3). Given the relative impact of learning curve estimates to the overall estimate, the research objective is to investigate the efficacy of the more common existing modeling techniques based upon the power function as well as compare the relative performance of those models to a sigmoid function. The specific investigative questions are as follows:

1. How well do power function based models perform relative to empirical data?
2. How well does a sigmoid function perform relative to the same data as above?
3. How do the common variants of the power function based models compare to one another as well as to the sigmoid function?
4. Does contemporary learning curve modeling methodology adequately explain the cost the cost behavior observed in the data?

Methodology

The primary basis of analysis is a statistical regression quality of fit comparison between the power and sigmoid functions relative to a common empirical data set. The statistical measure of comparison will be the coefficient of determination. The data used was collected from a serial manufacturing process of identical outputs. The analyzed data was normalized such that the unit cost per production iteration was sequentially recorded. Both the power and sigmoid functions were fit to the data minimizing the total deviation

between the observed values and the functional output. The analysis was conducted with Microsoft Excel and the accompanying Solver add-in software. Basic model formulation was conducted with Excel as well as the creation of scatter plots with graphed overlays of the functional models. Solver was used to calculate the error minimizing coefficients for the power and sigmoid functions. After identifying the optimal solution, the coefficient of determination was calculated for each function to determine which had superior fit.

Assumptions and Limitations

Certain specifics of the production process were unknown when the data analysis was conducted. The data used was not self-sourced. It was collected and provided by the production process operators and thus necessitated certain assumptions. The final outputs from each production activity analyzed was homogenous (same product specification). Minor modifications might have been made to the output specifications which were not otherwise indicated, but were exceptional and immaterial to the analysis. No significant production breaks were observed unless the data specifically noted contrary. A break in production often results in some degree of lost productivity. The circumstances and the length of time will generally dictate the severity of loss. Finally, the production process itself is assumed to be stable with regards to the capital and labor employed. If the type or mixture of capital or labor were altered, the nature of the analysis changes as the process performance would be affected. The principal limitation of this study is the narrow depth of the production process data analyzed, and consequently that limits the extent to which conclusions can be generalized to other similar processes.

Implications

If the research hypothesis is correct that a sigmoid function more precisely models the cost behavior of serially manufactured items than a power function, it will offer DoD cost analysts an effective tool for improving overall estimate accuracy. The typical DoD acquisition program has on average a final cost 46% in excess of its milestone B estimate. This estimative margin of error is unacceptably large to be useful for a resource manager. The learning curve component of an estimate is often significantly influential. A better estimating methodology could improve future cost estimate accuracy and enable better informed decisions for resource managers.

Preview

The next chapter will explore previous research and philosophy on the effect of process improvement on the unit costs of a serially manufactured product. There remains debate among scholars regarding certain particulars of process improvement. Particularly the long-term behavior of process improvement, work stoppage effects, and the impact of product specification changes. The third chapter will provide specifics of those analytical methods and processes employed for this research. The fourth chapter summarizes all the data analyzed and describes the key particulars identified. The fifth and final chapter will highlight key findings and conclusions from research overall.

II. Literature Review

Introduction

Learning is the accumulation of knowledge, understanding, or skill resulting from study, instruction, or direct experience. A learning curve is the two-dimensional graphical representation of the relationship between a quantitative measure of task performance and time. Performance is measured along the vertical axis and time on the horizontal axis. In finance and economics, performance is often a cost measure such as the number of labor hours required to manufacture a product. The horizontal axis (time) is either a continuous amount when engaged in the activity of interest measured from a specific starting point or discrete trials representing a specific production task or particular unit of production. The performance measure is not necessarily a direct function of time as implied by the graph, but is often influenced by numerous different factors. Typically, learning curves are used to assist with understanding and estimating the effect of those various influencing factors on performance. The analytical task is understanding and reasonably well segregating the influencing factors from each other to independently determine its effect on performance. The goal of learning curve analysis is to understand the impact of learning (accumulation of relevant knowledge and experience) on task performance. The implicit assumption is that relevant learning will accumulate from the initiation of an unfamiliar or foreign task and will be positively applied to enhance measured performance.

Learning curves and learning affected performance analysis has been conducted in numerous different disciplines. The first formal recognition and development of what is

now learning curve analysis is credited to Herman Ebbinghaus. Ebbinghaus executed a human learning experiment testing memorization based upon a created fictitious language (Ebbinghaus, 1885:22-24). In 1897 William Bryan and Noble Harter published a research summary of time relative telegraph operator performance. Performance was measured in words per minute sending and receiving (measured separately) and time was recorded in weeks from the experiment initiation. A graphic visualization of their findings shows that performance increases were rapid early, but diminished and performance stabilized as the experiment progressed (Bryan and Harter, 1897:45-53). Generally, most learning analysis was done within the field of human psychology with the objective of better understanding the more influential factors for memorization and information retention. Learning studies have since been applied to various other disciplines. Likewise learning analysis is applied to groups and organizational performance as well. Dr. Theodore Paul Wright is credited with being the first to formally conduct learning analysis in a commercial enterprise. His 1936 research *Factors Affecting the Cost of Airplanes* considered human learning among numerous factors that influenced aircraft production cost. Presently, the learning effect is a common consideration among cost estimators, accountants, and operations management practitioners when estimating and evaluating production process performance.

Analytical Context and Framework

The objective of learning curve research and analysis is to identify the factors and attributes most influential to performance changes. In the field of psychology researchers are frequently interested in factors that influence learning rates and retention. In the fields

of finance and economics analysts are generally concerned with the impact of learning on resource productivity. In both psychology and economics, the underlying objective is the same – manage and direct the learning process towards a desired outcome. A psychology researcher might seek to improve teaching methodologies, where a financial analyst may assess the effect of learning on production facility profitability. In either case developing a basic visualization of the learning effect through a learning curve is prudent to assist the analytical processes and develop estimative models.

Learning curve analysis within the discipline of finance or economics is generally applied to the analytical framework of a production process. A production process is the structural arrangement of resources (labor and capital) to transform some input(s) to some desired output(s) (Anupindi and others, 2012:3-6). Labor is the human workforce staffed in a production process and can be divided into direct and indirect labor. Direct labor is defined as those persons whose efforts can be traced to any one particular unit of output. Indirect labor includes the various support functions such as maintenance, engineering, and management. Capital is the firm's fixed infrastructure, machinery, and equipment required for production. A firm or organization can fully encompass a single production process, or there may be multiple production processes within a firm. A financial analyst or a cost estimator is tasked to understand and reasonably project the impact of learning on the production process.

Associated with the production process are certain performance measures that are essential to its effective management. Throughput is the total units of output produced in a standard amount of time (days, weeks, months, etc.) (Anupindi and others, 2012:55-58). Productive resources (labor and capital) likewise have an associated economic cost that is

measured per the standard unit of time. The ratio of economic cost for a specific resource, resource group, or the entire production process to corresponding throughput is economic cost per unit of output. Throughput is managed by changing the quantity, quality, or type of productive resources. The various combinations of those productive resources and the resulting throughput constitutes the production function for the process in question and is often expressed as a mathematical function such as Cobb-Douglas (Baye, 2010:156-165). This interpretation generally assumes that productivity levels of resources employed by a production process are relatively constant. Frequently, however, this is not true. Resource productivity levels often fluctuate depending upon the circumstances, and performance is affected by numerous factors. Learning has great potential to affect resource productivity. Learning is of great interest to process managers as it presents an opportunity to improve resource productivity without requiring additional investment. Distinguishing the changes in resource productivity consequent to learning effects and resource changes is critical to the correct analysis of a production process.

Mathematical modeling is used extensively in performing learning curve analysis. A mathematical model is the representation in mathematical terms of the relevant features and the behavior of some real-world phenomena (Bender, 1978:1). Within the context of an economic production process a learning curve is mathematically modeled such that the dependent variable (output) is the per unit production cost and the independent variable is either continuous time starting from absolute reference or the discrete count of production iterations. The most frequently used mathematical function for this purpose is the power function, shown in its basic form in Equation 1 and corresponding graph in Figure 1. Although unit cost is not a direct function of time it is modeled as such due to the implicit

relationship with the learning effect. Learning accumulates with time and its positive application is believed to reduce production costs. Learning itself is a function of time resulting from the exposure, experience, and increasing knowledge of a specific production process or technique. The critical assumption with this methodology is that learning is the only significant influence on unit cost. If other factors significantly affect cost, it compromises the learning analysis and another modeling methodology would be more appropriate. Essentially the modeling process devolves into curve fitting – identifying a mathematical function that has the best fit to a set of data points. Curve fitting is useful for describing and estimating behavior in a data set but the function which best fits the curve might not necessarily have a systemic relationship with the data it describes. A model that includes the significant variables that govern the dependent variable (unit cost) is preferred because it encapsulates the systemic relationship of the production process and would better explain and predict outcomes.

Development of Learning Theory

Theodore Paul Wright is credited with being the first to apply learning curve analysis to an economic production process with his research *Factors Affecting the Cost of Airplanes* published February 1936. Wright conducted his research within the Curtiss-Wright Corporation. Curtiss-Wright is a large aerospace corporation founded in 1929 and at the time of publishing was the largest nationally of that type. Wright's overall research objective was to improve the understanding of those significant factors most influential to aircraft production cost. Wright specifically wanted to develop a heuristic to describe and

estimate aircraft unit production cost. Wright listed numerous important categorical areas including tooling, specification changes, aircraft size, and batch production quantity. The central finding of Wright's research was a cost quantity relationship shown in Equation 3. This formulation has since become the basis of contemporary learning curve modeling.

$$y = ax^b \quad (3)$$

Where

- y = average cost per unit of output*
- a = estimated cost to produce a single unit*
- x = total number of units to produce*
- b = $\ln(1 - \text{cost reduction\% per doubling})/\ln(2)$*

Wright described this formula as a cost quantity curve that is used to compare the cost of a completed airplane in different quantities (Wright, 1936:125). A cost estimator would input the total number of aircraft for a single production run, the expected cost of a single unit, and the expected percentage cost decrease per quantity doubling. The result is the expected average per aircraft unit cost. Wright indicates that he began developing the cost quantity relationship in 1922. The curve initially began from two or three data points of cost quantity pairings for identical aircraft production and was later supplemented with additional data when it became available (Wright, 1936:122). Wright did not necessarily intend that his cost quantity relationship be used as a function for pricing individual units as either an independent variable of time or incremental unit production. Wright does not provide explicit detail of the internal specifics of production methods or production scale. Nevertheless, individual unit cost within a production run of a specific quantity will differ from those of another production run of differing quantity because the production process design would differ among batch sizes. Interpreting Wright's cost quantity relationship as a continuous function ignores the scaling limitations of a production process. Even within

the scope of the cost quantity relationship the limitations to scale a production process are evident with increasing quantities. Wright acknowledges this long-term possibility stating that percentage rate reduction of cost gradually declines with increasing production totals (Wright, 1936:125-126). Notwithstanding its limitations, Wright's curve has since been transformed and used as a continuous function for measuring individual unit cost. The formulation shown in Equation 3 has become the most common method for measuring cost decreases thought to be the consequence of learning. Several adaptations have since been made based upon the original formulation and have since been applied in numerous applications.

The most typical simplification of the learning curve is that increased cumulative output results in decreased average unit cost. However, the lingering question is why may that be the case. The most commonly offered explanation is that with each additional unit of production task familiarity and knowledge of the production process increases and thus allows for the identification of cost reducing improvements. This brings to focus the key difference between how Wright presented the learning curve and how it is generally used. From the perspective of a cost quantity relationship as determined by Wright the unit cost differential from one output quantity to another is predominantly a function of scale. That the selection of a specific production quantity would also imply the selection of a certain production process design, and the key difference being the process throughput and per unit average total cost. The alternative interpretation does not necessarily consider effects from the production process scale but implies that marginal cost reductions from one unit to the next is a predominant function of learning. Wright does not describe the particulars of the production process prevalent in his analysis, but suggests the process would change

depending on desired production quantity (Wright, 1936:124-126). Wright provides three general explanations for the behavior of the cost quantity curve. Wright reasons that labor learning, economies of scale, and resource selection impact the curve (Wright, 1936:124). Wright goes on to say that economies of scale is the principal factor of the three.

Learning Based Improvement

Labor learning is the most frequently offered reason for the expected behavior of learning curves and one of three reasons mentioned by Wright when developing the cost quantity curve. Wright specifically states that “improvement in proficiency of a workman with practice and particularly if time in motion studies are made, is well known” (Wright, 1936:124). Often, a large part of the cost reduction in a production process is believed to originate from improvements in the direct labor force. This thinking was confirmed to an extent by the empirical study conducted by Dr. Nicholas Baloff in which he found a gradual decrease and an eventual flattening of the learning curve in a capital-intensive production process, but found no such decrease or flattening in the labor-intensive processes studied (Yelle, 1979:310). The common understating was that a human paced production process will likely show more cost improvement than one paced by machines given the human capacity to learn and improve (Hirschmann, 1964). Additionally, the results of human task performance studies ranging from the earliest such as those of Ebbinghaus and Bryan and Harter to modern studies are supportive. The general belief is that individual performance improvement in the context of a production process will aggregate and accrue to the firm enhancing overall productivity. However, other comprehensive studies of the firm offer a

different narrative. Dr. Kazuhiro Mishina conducted an analysis of Boeing B-17 bomber production during World War Two. The study focal point was Boeing Plant Number Two in Seattle Washington. With respect to labor learning Mishina identified that direct labor learning did not play as significant a role as once thought in improving overall production productivity. He found that workers generally were not particularly skilled or experienced in aircraft production and turnover was high disallowing for much in the way of learning (Mishina, 1999:162-163). The overwhelming majority of the productivity gains observed in the plant was a function of managerial improvements to the overall production process and not improvements of direct labor proficiency (Mishina, 1999:164). Mishina states that management's resource employment and organization decisions rather than gains in proficiency of the resources themselves accounted for the overall success of the plant. Similar findings were noted in a study of a truck assembly plant conducted by Dr. Dennis Epple. The objective was to compare the learning gains of two separate production shifts which at one point had operated as a single shift. He found that both shifts were equally productive and that gains in learning are embedded in the organizational structure as well as its technology. He also identified that the second shift had a reduced improvement rate compared to the first because of lowered managerial and industrial engineering oversight (Epple and others, 1996:84-85). This research emphasizes the importance of management and structural arrangements within the firm as it pertains to its overall productivity. Those gains from direct labor learning are not as significant a contributor to organizational gains when compared to the resource employment and production process design choices.

Economies of Scale

The second explanation offered by Wright for the underlying behavior of the cost quantity relationship is economies of scale. Economies of scale exists when average total costs decline as productive output (throughput) is increased (Baye, 2010:185). Logically, this is congruent with the cost quantity interpretation of Wright's report, that unit cost is predominantly a function of the production process design or subsequent changes thereto. The cost quantity relationship implies that for higher expected total production quantities a production manager would choose to produce on the most optimal point on the long run average total cost curve that does not exceed the total amount and remains within the boundaries of productive resource constraints.

Wright indicates that per unit capital and labor costs will decline as a function of increasing scale. Wright specifically states that increased scale allows for the economical substitution of labor for capital, reduced setup costs for both labor and capital, and a more efficient spread of indirect (overhead) cost (Wright, 1936:124-126). Mishina's research supports this finding. His B-17 production process analysis indicates that the cost reductive learning which occurred was primarily the result of management decisions and not increased direct laborer proficiency. Mishina indicates that B-17 direct hour labor requirements declined from approximately 71 worker-years to 8 worker-years from 1941 to 1945 (Mishina, 1999:150-151). Additionally, he identified that the predominant reason for the observed improvement, particularly from the early phases of the program, resulted from increasing production scale (Mishina, 1999:175-176). Furthermore, the majority of aircraft labor cost reduction occurred during the operations scale-up phase occurring from

approximately May 1940 until December 1942 (Mishina, 1999:159). During this period, Boeing emplaced 91% of the total fixed investment in terms of Plant Number Two floor space, established the Tooling Department (managed 70,000 dies and jigs), and increased the total direct labor force from 9,972 employees in August 1941 to 17,000 in February 1942 (Mishina, 1999:157, 162).

In a comparative analysis between Boeing Seattle, WA and Ford Willow Run, MI (B-24 Liberator), Mishina notes the respective differences between their learning curves. Mishina specifically indicates that the Willow Run learning curve stabilized more rapidly than Boeing Seattle. He attributes this to the process design difference. Ford used a more capital-intensive design where Boeing had a labor-intensive focus. Mishina described the Ford Willow Run learning curve as a period of necessary adjustment before the process achieved its total designed potential (Mishina, 1999:167-168). This corroborates previous research by Baloff who identified that the learning curves of capital-intensive production processes plateaued in 75% of the cases studied where the labor-intensive processes did not (Yelle, 1979:310-311). It is possible that Baloff, like Mishina, was observing the higher relative flexibility of a labor-intensive process over a capital-intensive one. Research from Dr. Peter Thompson identified similar findings regarding investment and production scale increase. Thompson analyzed World War Two Liberty Ship production with a focus on seven of the largest producing shipyards at the time. Thompson found a positive relationship between capital investment and labor productivity levels. Additionally, there is also a positive relationship between periods of increasing capital investment, productivity growth, and increasing throughput (Thompson, 2001:121-122). Thompson concludes that traditional learning curve analysis suffers from an omitted variable bias.

Specifically, with respect to Liberty Ships he found that omitting the capital investment component overstates the contribution of learning to productivity increases (Thompson, 2001:132).

Given the aforementioned empirical studies the possibility exists that the typical visualization of decreasing unit costs concurrent with increasing production quantities is not necessarily a function of incremental learning but instead represents the modification of the production process as it moves to a more efficient location on the average cost curve. The traditional method of modeling learning analysis does not develop a systemic relationship to those other major underlying contributing factors which potentially results in misunderstanding the behavior of a production process.

Resource Selection

The third and final component is resource selection. The nature and design of the production process alters labor force expertise requirements. Generally, the resource cost of production workers is measured with time (hours typically) and not absolute economic cost. This is appropriate for long term comparative purposes as it discounts for wage rate fluctuations and inflation. However, it does not show the cost impact of selecting labor of differing skill classes. Wright mentions the association of the production process design, economies of scale, and resource selection. Specifically, he notes that increased production scale, supplementing or supplanting labor with capital, and procedure standardization will reduce the need for highly skilled or specialized labor reducing costs (Wright, 1936:124). Mishina states that construction of the first B-17s in 1935 was done primarily with highly

skilled workers using hand tools (Mishina, 1999:157). This highly contrasts with the later scaled-up production operations when Boeing employed large numbers of workers who on balance had little if any manufacturing experience or knowledge (Mishina, 1999:163). In this situation, an adequate pool of experienced craftsman was not available. However, the production process transformation from low volume to standardized high volume did not require such skilled labor and consequently wage rates declined lowering average unit labor cost.

Summary and Conclusion

The most common interpretation of the learning curve when applied to production processes is that increased production, measured in either incremental units or time, will increase the relevant knowledge and experience of the organization, and though learning effects reduces unit costs with time. The research of Theodore Wright's *Factors Affecting the Cost of Airplanes* is near universally attributed as the basis of this understanding. However, his research indicates that the cost quantity relationship which he developed was not derived from different cost quantity observations of a single production process. Instead it was developed from the cost quantity observations from distinctly different production processes of the exact same output. Although Wright did not provide the particular details of the production processes he surveyed, it is reasonable to deduce that each process was designed to optimally produce at differing levels of throughput. From this purview, the cost quantity relationship presented by Wright is likely a feature of economies of scale, and not necessarily attributable to the incremental benefit of learning. Furthermore, Wright verifies

this understanding stating “one of the principal factors is the economy of labor which greater tooling can give as the quantity increases” (Wright, 1936:124). This would imply that the typical visualization of decreasing unit costs with increasing quantities that accompanies the typical learning curve scatter chart is not necessarily the result of passive learning but the real-time effect of production scale-up (moving from low to high volume production). The implication is that a true model of production process performance would require multiple independent variables as opposed to the common single independent variable, and that present modeling methodology is non-systemic with regard to the underlying influencing factors and is better characterized as extrapolated curve fitting.

III. Methodology

Introduction

The objective of the methodology is the development of an unbiased quantitative and qualitative analysis of serial production process cost performance data. The purpose is evolving an improved understanding of the effect of learning on cost performance, and to improve the estimating methods thereof. Probabilistic mathematical modeling is the central method of analysis to develop the results. Field data was collected from several different serial production processes. Final results are used to further develop insight and appreciation for the learning effect in a production process.

Tools and Equipment

Spreadsheet modeling and nonlinear optimization were the principal means which the analysis was accomplished. Microsoft Excel 2016 installed on a Microsoft Windows 7 desktop PC was the platform used. The Solver add-in software from Frontline Systems for Excel was used to accomplish the nonlinear optimization calculations. The pairing of Excel and Solver allows for simple development of probabilistic mathematical models as well as accompanying data visualizations such as scatterplots, box plots, line graphs, and histograms. The Real Statistics Resource Pack add-in for Excel was also used to automate the calculation of certain statistical measures. High functionality and high relative ease of use compared to similar alternatives prompted these decisions.

Field Data

The field data selected for analysis in this research was sourced from varied serial production processes. The data was collected by the process owning firm or organization. Data collection methods and systems were automated and manual. The data are assumed highly reliable as it is used by the firm for its internal process management decisions and externally for billing and compliance. Each data set varied but at minimum each had a trial number and cost performance measure. Each unit of output is produced in sequence and assigned an ordinal number starting with the initially completed unit and continuing until the process is terminated. This sequence or trial number is often augmented with the date and time of completion. The measure of cost performance in all the data analyzed was the labor time required for task completion. Other quantitative and qualitative measures such as production breaks or product variants were available but differed from one data set to the next. Data recording generally started with production process initialization particular to a specific output.

Analytical Method

Probabilistic mathematical modeling is the central analytical technique employed in this process. A probabilistic model or function is one that has a deterministic aspect as well as a random error component (McClave and others, 2014:603). The functions used to model the data are the basis of the deterministic component. The random error is found through the regression analysis techniques. The regression process is designed to identify

those functional parameters which minimize the error or total deviation between function output and the corresponding observation (McClave and others, 2014:607). The cost data, collected from the various production processes, is being regressed with the experimental functions to assess their suitability for modeling and estimating cost performance. Data is visualized with a scatterplot to help develop a rudimentary behavioral assessment and for the consideration of certain qualitative factors. Finally, a review of regression quality and the validation of certain key assumptions is performed.

The first task in executing the analytical method was to review the field data and to standardize it for insertion into the modeling template. Specifically, the relevant time values and the cost performance measures were reviewed to ensure conformity with the spreadsheet standard. Following a review of the data it was then placed into the modeling template for analysis. A segment of example data is shown below in Figure 3.

<i>No.</i>	<i>Group</i>	<i>Item</i>	<i>Date</i>	<i>Hours</i>
1	EMD	91-A4001	08/08/97	33614
2	EMD	91-A4002	07/25/98	22667
3	EMD	91-A4003	02/24/00	28592
4	EMD	91-A4004	10/31/00	25159

Figure 3. Spreadsheet Modeling Template (Example Data)

Once the data was placed into the modeling template the regression process commenced. The principal method for accomplishing this was through the complimentary Solver Excel add-in software. Solver was used to minimize the total sum of squares of the errors (SSE) for each experimental function. The error term or residuals are the difference between the observed value (hours) and the value calculated by the model which corresponds to the

same independent value (item number). The errors are squared and summed to calculate SSE. The principal basis of regression analysis is identification of those parameters that minimize the SSE for a mathematical function (McClave and others, 2014:607). Solver was utilized to minimize SSE by adjusting the corresponding parameters using nonlinear optimization. Figure 4 presents an example of the Solver interface.

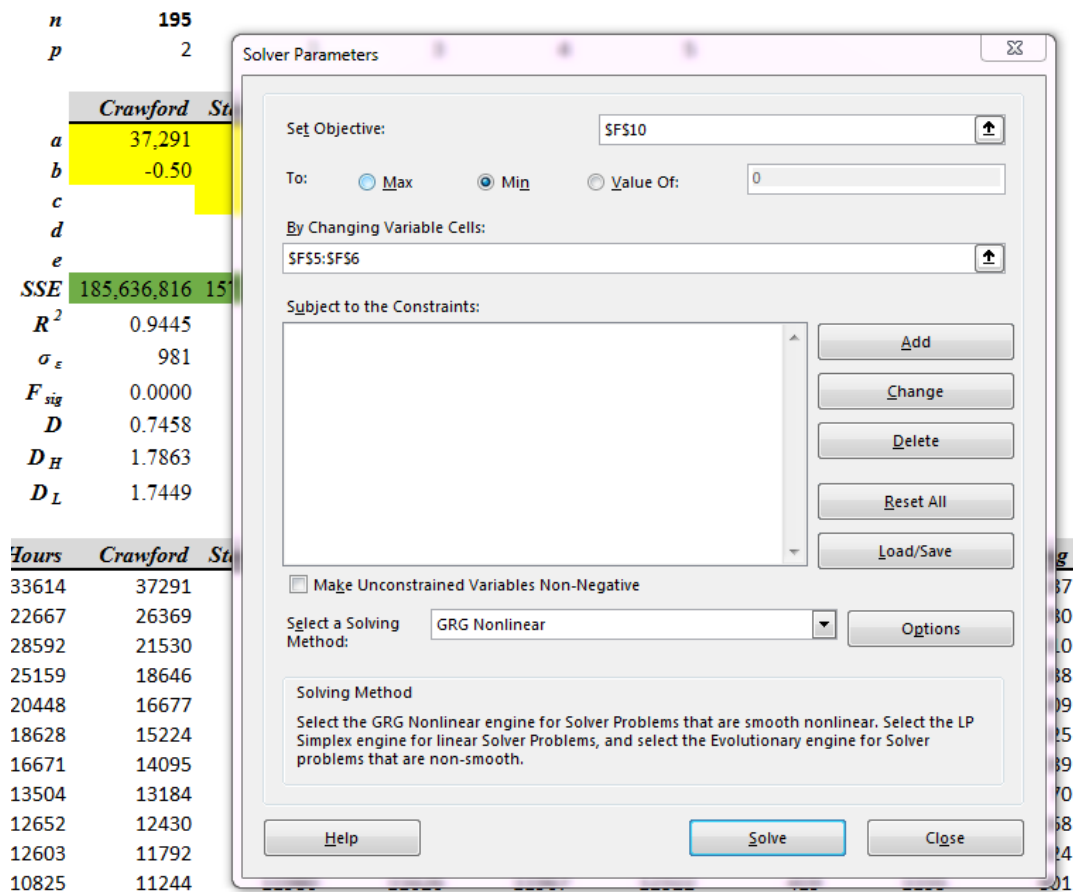


Figure 4. Spreadsheet Modeling Template (Solver Interface)

In total five univariate mathematical models were tested for each data set. Four of the five functions are variants of the base form power function first shown in Equation 1 and graphed in Figure 1. The fifth function is the sigmoid function (s-curve) displayed in

Equation 2 and graphed in Figure 2. The cost quantity relationship originally proposed by Wright is the basis of the power function based models. Several variants have since been created with the objective of developing a more robust model. Four of them were tested as part of this research. The first is the reinterpretation of the Wright cost quantity curve to a unit cost curve generally attributed to James R. Crawford (AFCAH, 2007:4). Rather than the independent variable being the total production quantity and the output variable the average total cost to produce that amount, the unit curve interpretation states that the independent variable is an ordinal production unit and the output variable is the cost of that particular unit. The unit theory is shown in Equation 4.

$$y = ax^b \quad (4)$$

Where

$$\begin{aligned} y &= \text{function output (cost)} \\ x &= \text{independent variable (unit number)} \\ a &= \text{constant (first unit cost)} \\ b &= \text{exponential constant (learning)} \end{aligned}$$

The second is the Stanford-B model which adds an additional constant which allows for the lateral shifting of the curve along the horizontal axis (Badiru, 1992:178). This added variable allows for increased ability to fit the curve to any given set of data compared to the basic unit curve. Shown in Equation 5.

$$y = a(x + b)^c \quad (5)$$

Where

$$\begin{aligned} y &= \text{function output (cost)} \\ x &= \text{independent variable (unit number)} \\ a &= \text{constant (first unit cost)} \\ b &= \text{lateral shift (prior units of experience)} \\ c &= \text{exponential constant (learning)} \end{aligned}$$

The third is the De-Jong learning formula. This variant introduced the incompressibility factor (M) concept to the unit theory curve. Incompressibility is the percentage amount (ranging from 0 to 1) of the production process that is machine automated (Badiru, 1992: 178-179). A negative exponent basic form power curve approaches zero on the vertical axis as the independent values approach positive infinity. Within the context of a learning curve this implicitly means that costs will effectively approach zero with increased unit production. The incompressibility factor essentially places a floor beneath the learning curve and in the context of the DeJong model represents the unchanging nature of machines beneath infinitely compressible unit labor costs. Mathematically this allows the unit theory curve to be shifted vertically, potentially allowing for improved fit to a data set compared to the unit curve. The mathematically transformed DeJong model is shown in Equation 6.

$$y = a + bx^c \quad (6)$$

Where

- y = function output (cost)*
- x = independent variable (unit number)*
- a = constant (capital operation cost)*
- b = constant (first unit labor cost)*
- c = exponential constant (learning)*
- C = a + b (first unit total cost)*
- M = a ÷ C (incompressibility factor)*

The fourth and final variation of the power curve that was tested as a part of this research is the combination of the Stanford-B and the DeJong model attributed to Gardner W. Carr (Badiru, 1992:178). Mathematically this variant provides full control over the positioning of the curve in two-dimensional space maximizing the possibility for fitting to a data set. Shown in Equation 7.

$$y = a + b(x + c)^d \quad (7)$$

Where

y = function output (cost)
x = independent variable (unit number)
a = constant (capital operating cost)
b = constant (first unit labor cost)
c = constant (prior units of experience)
d = exponential constant (learning)
C = *a* + *b* (first unit total cost)
M = *a* ÷ *C* (incompressibility factor)

Lastly, the sigmoid function or s-curve was tested. The s-curve is most similar to the Carr model of the power curve family. The shape of the curve can be directly controlled, and it can be placed anywhere in two-dimensional space as well. Shown in Equation 8.

$$y = a + \frac{b}{(1 + e^{c(x+d)})^e} \quad (8)$$

Where

y = function output (cost)
x = independent variable (unit number)
a = constant (lowest value)
b = constant (upper most in excess of lowest value)
c = constant (curve saturation point)
d = constant (lateral shift)
e = constant (curve symmetry control)

The quantitative measures and qualitative indicators of fit quality are reviewed for each function following the identification of its SSE minimizing parameters. Regression summary statistics are reviewed to assess quantitative fit quality. Specifically, the F test for overall model validity is reviewed, adjusted coefficient of determination (R^2), and the standard error of the regression for each function. The F test is used to determine if the regression is statistically significant to an σ level of 0.05. Adjusted R^2 is a measure of the functions explanatory power relative to the data set (McClave and others, 2014:634-636). The standard error of the regression is the standard deviation of the error (McClave and

others, 2014:619-620). Valid R^2 values range from zero to one with one being a perfect explanatory model. The standard error of regression value can range from zero to positive infinity with zero being perfect predictability. Sample output is shown below in Figure 5.

	<i>Crawford</i>	<i>Stanford-B</i>	<i>DeJong</i>	<i>Carr</i>	<i>Sigmoid</i>
<i>a</i>	37,291	47,720	-440	2,722	3,870
<i>b</i>	-0.50	1	37,241	245,850	36,119
<i>c</i>		-0.56	-0.47	5	1.90
<i>d</i>				-1.19	1.74
<i>e</i>					0.06
<i>SSE</i>	183,926,140	157,149,378	183,158,206	111,662,669	225,136,399
R^2	0.9450	0.9528	0.9450	0.9663	0.9317
σ_ϵ	976	905	977	765	1,089
F_{sig}	0.0000	0.0000	0.0000	0.0000	0.0000
<i>D</i>	0.7499	0.8686	0.7393	1.3078	0.7033
D_H	1.7863	1.7969	1.7969	1.8076	1.8184
D_L	1.7449	1.7345	1.7345	1.7239	1.7133

Figure 5. Spreadsheet Modeling Template (Summary Statistics)

One of the objectives of this research was to determine if there was any trend within the data sets with regard to one model being statistically favored over the other. In addition to the adjusted R^2 values, a pairwise analysis of the variance (ANOVA) test was performed to determine which of the models was preferred to a statistical significance level of 0.05. This test is useful to determine if one particular model is preferred to another as adjusted R^2 values alone may not necessarily be indicative (Motulsky and Ransnas, 1987:371). Sample output from this test is shown in Figure 6.

	<i>Crawford</i>	<i>Stanford-B</i>	<i>DeJong</i>	<i>Carr</i>	<i>Sigmoid</i>
<i>Crawford</i>	0.5000	0.8620	0.5115	0.9997	0.0813
<i>Stanford-B</i>	0.1380	0.5000	0.1448	0.9907	0.0067
<i>DeJong</i>	0.4885	0.8552	0.5000	0.9997	0.0773
<i>Carr</i>	0.0003	0.0093	0.0003	0.5000	0.0000
<i>Sigmoid</i>	0.9187	0.9933	0.9227	1.0000	0.5000

Figure 6. Spreadsheet Modeling Template (Comparative ANOVA Test)

The primary qualitative indicators of fit quality assessed are the scatterplot with function overlay and the corresponding residuals scatter plot. Although statistical tests are required for validating the results of a model, visual inspection of the data can provide additional insight to the behavior of the data. Sample output shown in Figure 7.

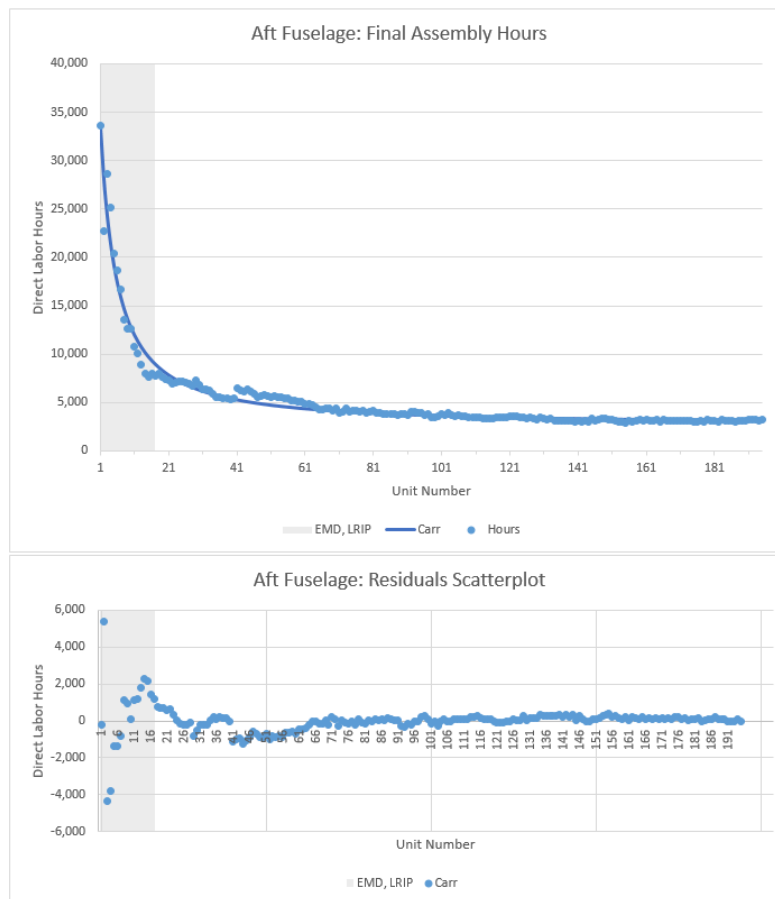


Figure 7. Spreadsheet Modeling Template (Data and Error Scatter Plots)

The remaining analytical method component is regression assumption validation. Regression analysis is just as much if not more a qualitative effort than a quantitative one. Without sound, qualitative analysis of the output the final results are possibly invalid and unreliable. Three qualitative tests are performed to determine regression quality and are as follows: error normality, homoscedasticity, and autocorrelation. In regression analysis, the mean of the total errors is zero and generally is assumed to be normally distributed. A departure from normality signals a possible significant problem with the analysis (Evans, 2013:274). Normality requires the data points are normally distributed around the center point of the modeled function. Normality was determined with a Shapiro-Wilke statistical test. Additionally, the box plot for each function was reviewed as well. Homoscedasticity is the second assumption and requires the consistent distribution of errors throughout the entirety of the data set. Levene's statistical test was used to test for homoscedasticity. The residuals scatterplot was inspected as well. A homoscedasticity violation is indicative of a significant problem in the regression model (Evans, 2013:274). Autocorrelation, the third assumption, requires that errors should be independent of each other. A departure of error independence, particularly with time sequenced data, indicates a significant problem, and that an alternative method should be considered (Evans, 2013:274). Error independence was determined with a Durbin-Watson statistical test. The scatterplot of errors was also examined as well. These assumptions must be considered in addition to the quantitative and qualitative factors to make an overall determination of regression validity. Methods used to assess the assumptions of regression are summarized in Table 1.

Table 1. Summary of Regression Assumptions Validation Tests

Normality	Homoscedasticity	Autocorrelation
Shapiro-Wilke	Levene's Test	Durbin-Watson
Box Plots	Error Scatterplot	Error Scatterplot

Results

Following field data analysis completion, the primary output is the data scatterplot with an overlay of the modeled functions, R^2 measures, and standard errors of regression. Complimentary outputs are the scatterplots for errors and error box plots.

IV. Data Description and Analysis

Introduction

In the context of an economic production process, learning curve analysis is used to enhance the understanding of production cost behavior over time, as well as estimating future behavior. If a mathematical learning curve function is formulated that can establish a systemic relationship with production process cost behavior, then it could be reasonably utilized to describe and estimate future cost behavior. However, if a systemic relationship cannot be established then the mathematical function could be used only as a fitted curve, and applied on discretionary basis for description and limited extrapolation. The allure of learning curves is that production process cost behavior could be explained with only one controlling independent variable. The reality however is that numerous significant factors influence production costs which disallows robust single independent variable modeling. Learning analysis generally has been a best fit curve analysis process which explains the creation of the numerous competing models and methods over time. The overall research objective is to develop actionable insight using available cost data to enhance production process cost estimating methodologies. The specific research questions, first presented in Chapter 1, are shown again below:

1. How well do power function based models perform relative to empirical data?
2. How well does a sigmoid function perform relative to the same data as above?
3. How do the common variants of the power function based models compare to one another as well as to the sigmoid function?

4. Does contemporary learning curve modeling methodology adequately explain the cost the cost behavior observed in the data?

Sequential production cost data was analyzed from five production activities with the methods described in Chapter 3. Each of the five mathematical functions mentioned in Chapter 3 and summarized in Table 2 below were fit to the production process data. The results were reviewed to assess quality of fit as well as the relative fit quality between the different functions. Relevant supplementary information is included wherever appropriate to enhance the analysis.

Table 2. Summary of Models

Common Name	Form	Type
Unit Cost Model (Crawford)	ax^b	Power
Stanford-B Model	$a(x + b)^c$	Power
DeJong's Learning Formula	$a + b(x)^c$	Power
Carr 1946 ("S-curve")	$a + b(x + c)^d$	Power
Sigmoid Curve (S-Curve)	$a + \frac{b}{(1 + e^{c(x+d)})^e}$	Exponential

Air Force Advanced Tactical Fighter (ATF) Program

The Advanced Tactical Fighter (AFT) is a single-seat, twin-engine, all-weather, stealth, air superiority fighter aircraft exclusively operated by the United States Air Force (USAF). In total 195 ATFs were produced. The first ATF was delivered to the USAF on 9 April 1997 and the final on 24 April 2012. The initial nine aircraft were produced within the Engineering and Manufacturing Development (EMD) phase of the acquisition process. The remaining 186 aircraft were final production models produced thereafter. Corporation A was the prime contractor and Corporation B was a prime partner. Manufacturing

responsibilities for significant aircraft components and systems was segmented between the two. Corporation B was responsible for the wings, aft fuselage, avionics integration, 70% of mission software, training systems, life support, and protection systems (Boeing, 2014). Corporation A was responsible for program management, forward and center fuselage, control surfaces and stabilizers, and critical avionics systems (Boeing, 2014). Fabrication of certain components and final assembly for the aft fuselage and wings was completed at the Corporation B Integrated Defense Systems industrial center located in Seattle, Washington (Waurzyniak, 2005). The production process was subdivided into numerous activities and sub-activities which fed the respective final assembly tasks for the aft fuselage and wings. Completed units were delivered to Corporation A in Marietta, Georgia for integration and final assembly (Waurzyniak, 2005).

A learning curve analysis was performed for the requisite number of direct labor hours to complete those significant activities for a single aft fuselage or wing paring that had available data. Five major activities were analyzed which are as follows: aft fuselage final assembly, aft fuselage feeder line, wing final assembly, wing spar fabrication, and wing skins composite fabrication. Both aft fuselage and wing final assembly involved the integration of all requisite component parts for completion. The aft feeder line performed pre-assembly work for certain components and assemblies that were later integrated with the aft fuselage (“Lean on Me”, 2004). Wing spars are primary structural members that run lengthwise the wings. The wing skins are the outermost surface of the wing assembly. The data was sourced from DoD Cost Assessment and Program Evaluation (CAPE) and collected by Corporation B. The specific collection method and techniques employed by Corporation B is unknown. The fields within the data set are the group, aircraft number,

and direct labor hours. Group is a descriptive field indicating the production phase or lot that the aircraft belongs. Each aircraft is assigned a unique identification number of the form FY-A4xxx where FY is the contract fiscal year and the trailing three digits is a unique number. The exact method Corporation B used to account for total labor hours is unknown as well. The data was supplemented with information from F-16.net. F-16.net is a community open source data repository for military aircraft and was used to identify the aircraft delivery dates.

Regression analysis was performed on the five data sets utilizing the spreadsheet modeling template and methodology described in Chapter 3. Regression analysis for the five mathematical functions displayed in Table 2 was conducted for each data set for 25 in total. All of regressions for the tested functions were statically significant to an α level of 0.05 per the overall ANOVA test. The approximate range of adjusted R^2 values for all regressions is from 0.81 to 0.98. This information is summarized below in Table 3.

Table 3. Regression Summary Statistics

		Crawford	Stanford-B	DeJong	Carr	Sigmoid
Aft Fuselage	R^2	0.9445	0.9528	0.9450	0.9663	0.9317
	σ_ε	981	905	977	765	1,089
Aft Feeder Line	R^2	0.9237	0.9627	0.9392	0.9768	0.9678
	σ_ε	137	96	123	76	89
Wings	R^2	0.9316	0.9495	0.9341	0.9705	0.9561
	σ_ε	1,925	1,653	1,890	1,264	1,542
Wing Spar	R^2	0.8089	0.8554	0.8387	0.8560	0.8116
	σ_ε	122	106	112	106	121
Wing Skins	R^2	0.8852	0.8848	0.8846	0.8851	0.8350
	σ_ε	105	105	105	105	126

For each of the five activities analyzed, the power function based learning curve variant credited to G. W. Carr was either the superior choice or among the preferred alternatives. This was determined with comparative ANOVA testing between each of the alternatives to a statistical significance level of 0.05. For two of the five data sets this statistical test is indeterminate between the alternatives with the highest overall fit quality. A summary of this information is presented below in Table 4.

Table 4. Model Fit Quality - Preferred Alternative(s)

Aft Fuselage	Aft Feeder Line	Wings	Wing Spar	Wing Skins
Carr	Carr	Carr	Stanford-B	Crawford
			DeJong	Stanford-B
			Carr	DeJong
				Carr

All three of the qualitative tests mentioned in Chapter 3 which are required to validate the statistics of the regression analysis failed for each regression. The three testing categories reviewed were normality of the residuals, the absence of homoscedasticity, and error term independence or autocorrelation. The primary consideration for testing normality was the Shapiro-Wilk test and visual inspection of the respective error term box plot. The primary method used to test homoscedasticity was a Levene's test as well as a visual inspection of the error term scatter plot. Lastly, the Durbin-Watson test was used to detect the presence of autocorrelation as well as visual inspection of the error term scatter plot.

Despite the unanimous failure of all three tests for regression model assumptions, these departures do not invalidate the utility of the information or the results of the curve fitting process. Specifically, assumption violations impair the ability to develop measures of statistical reliability for the estimators (McClave and others, 2014:619). Nevertheless, the failures do indicate the presence of some problem within the model. A scatter plot of the data set for aft fuselage assembly is shown in Figure 8.

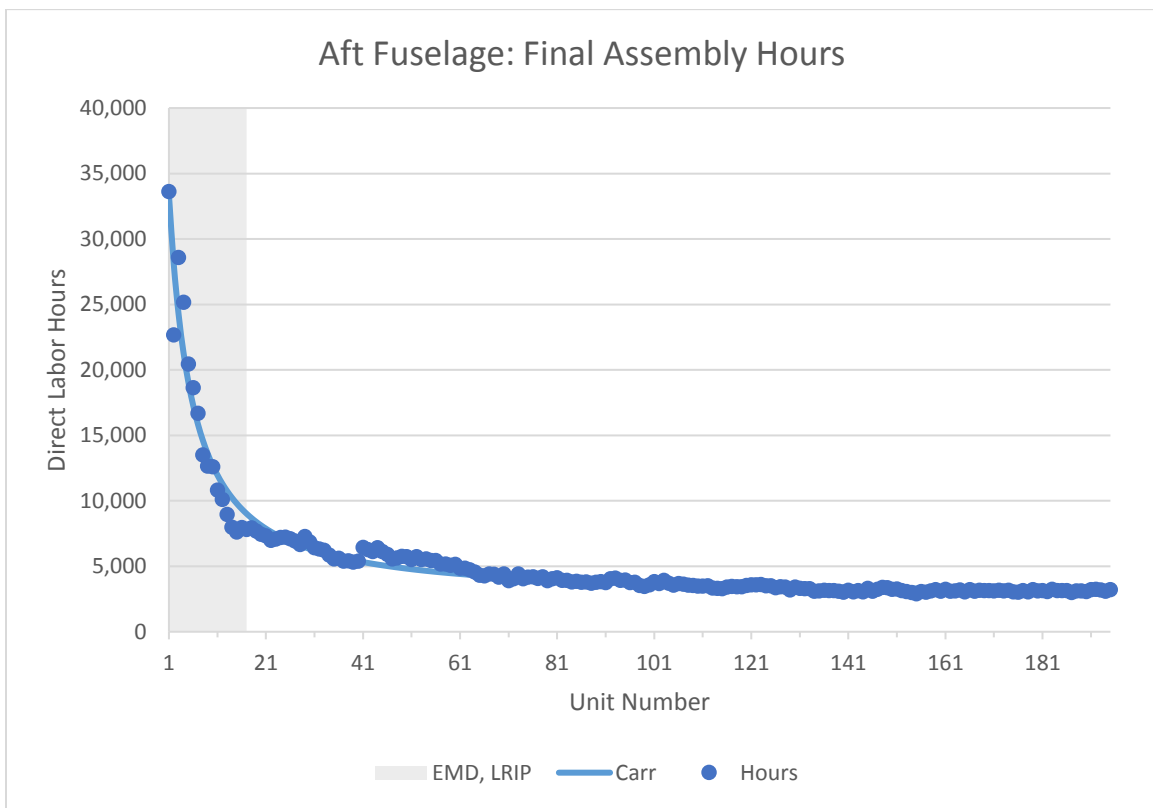


Figure 8. Aft Fuselage Final Assembly Hours (Carr 1946)

For the purposes of clarity only the Carr variant of the power curve family is shown. The Carr variant was the statistically preferred variant for the aft fuselage data set. Variability is generally present in every production process (Anupindi and others, 2012:231). Clearly

this is the case in the aft fuselage production process and not at all unexpected. The curve fitting process generally attempts to place the mathematical function directly through the center of the data set. Assuming a reasonable fit of the function to the data overall, as was the case in this situation, the observations very well may oscillate around the fitted trend line consequent to the variability in the production process. The residuals scatterplot for the Carr model for aft fuselage assembly shown in Figure 9 clearly displays this effect.

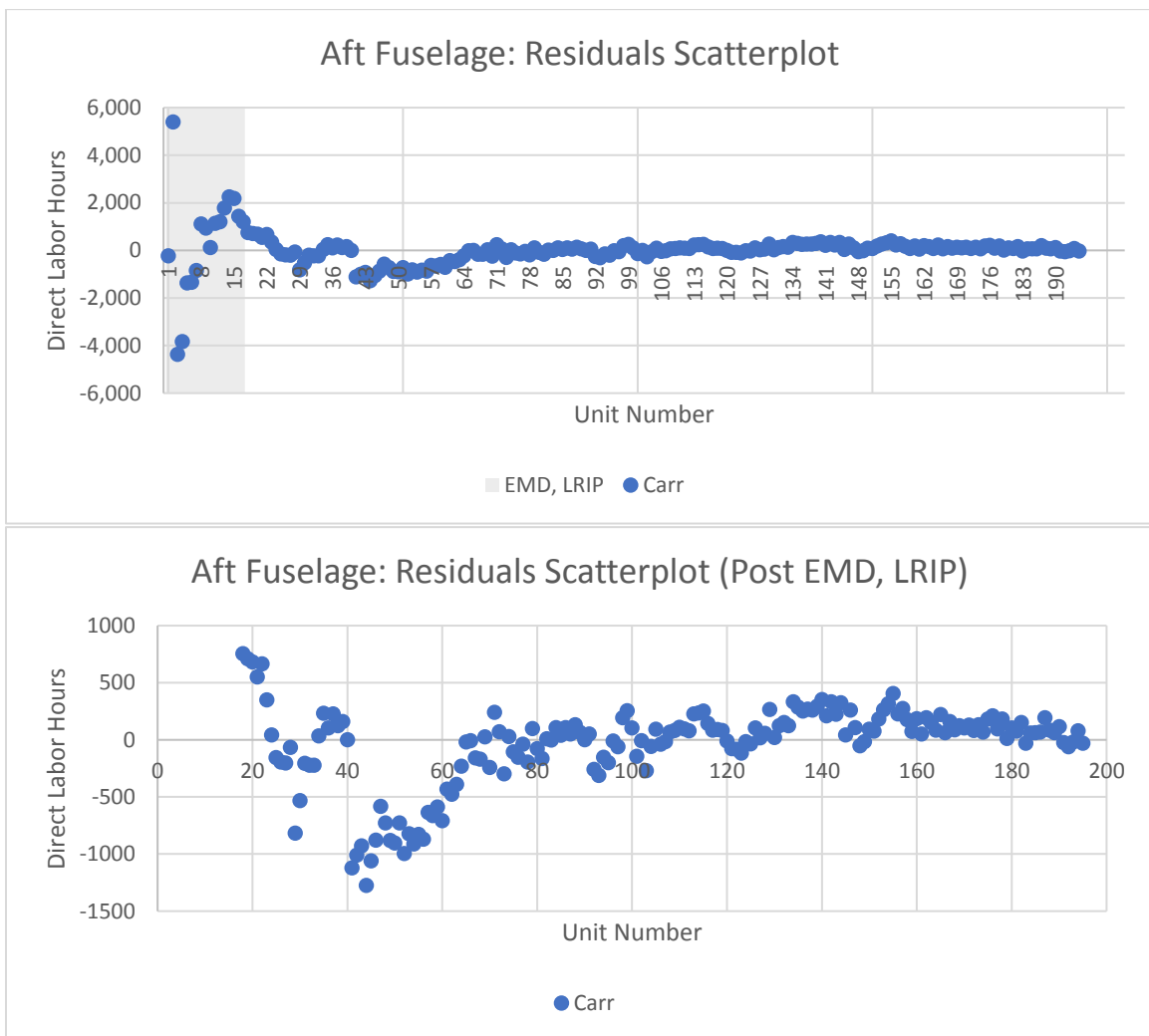


Figure 9. Aft Fuselage Assembly Hours Residuals Scatterplot (Carr 1946)

The upper panel of Figure 9 displays all residuals with the EMD and LRIP highlighted in grey. The lower panel shows just the production units for added clarity. Overall the errors show a heteroscedastic trend that lessens with increasing unit count implying variability was actively being removed from the production process and corroborated with Corporation B's embracement of lean production philosophies for the ATF program (Waurzyniak, 2005). Additionally, whatever caused the variability in the first place likely had some systemic relationship with time resulting in the oscillation pattern in the residuals and accounts for the autocorrelation and normality error given the uneven nature of the movement. These problems were prevalent in all production process regression models. While the overall pattern is understandable, it does not necessarily account for the wider range in variation for the EMD and LRIP compared to the remainder of the program. The disparity between the two segments is significant and would imply that the two segments, as they are shown in this presentation of the data, are somehow fundamentally different.

The initial hypothesis from the commencement of this research was that a sigmoid function would outperform a power function for modeling the data of production process. The results from the five processes analyzed as part of this research does not support this position. In each of the five production activity data sets one of the power function based variants was the preferred alternative. The predominant reason for this outcome is that the power function can generate curves that resemble a capital letter 'L' where the vertical and horizontal components of the L meet in a gradual curve as opposed to a sharp point. This shape is generally well suited to approximate the near vertical cost decline for early units in the EMD and LRIP phase as well as the production units. However, the power function does not perform well in situations where the process stabilizes, and incidentally was the

original motivation for this research. Four of the five production processes did eventually show signs of stabilization and it was most prevalent in the wing skins fabrication task. A scatter plot with plots for both the Crawford and Sigmoid functions are shown in Figure 10 and the Crawford curve error term scatter plot as well.

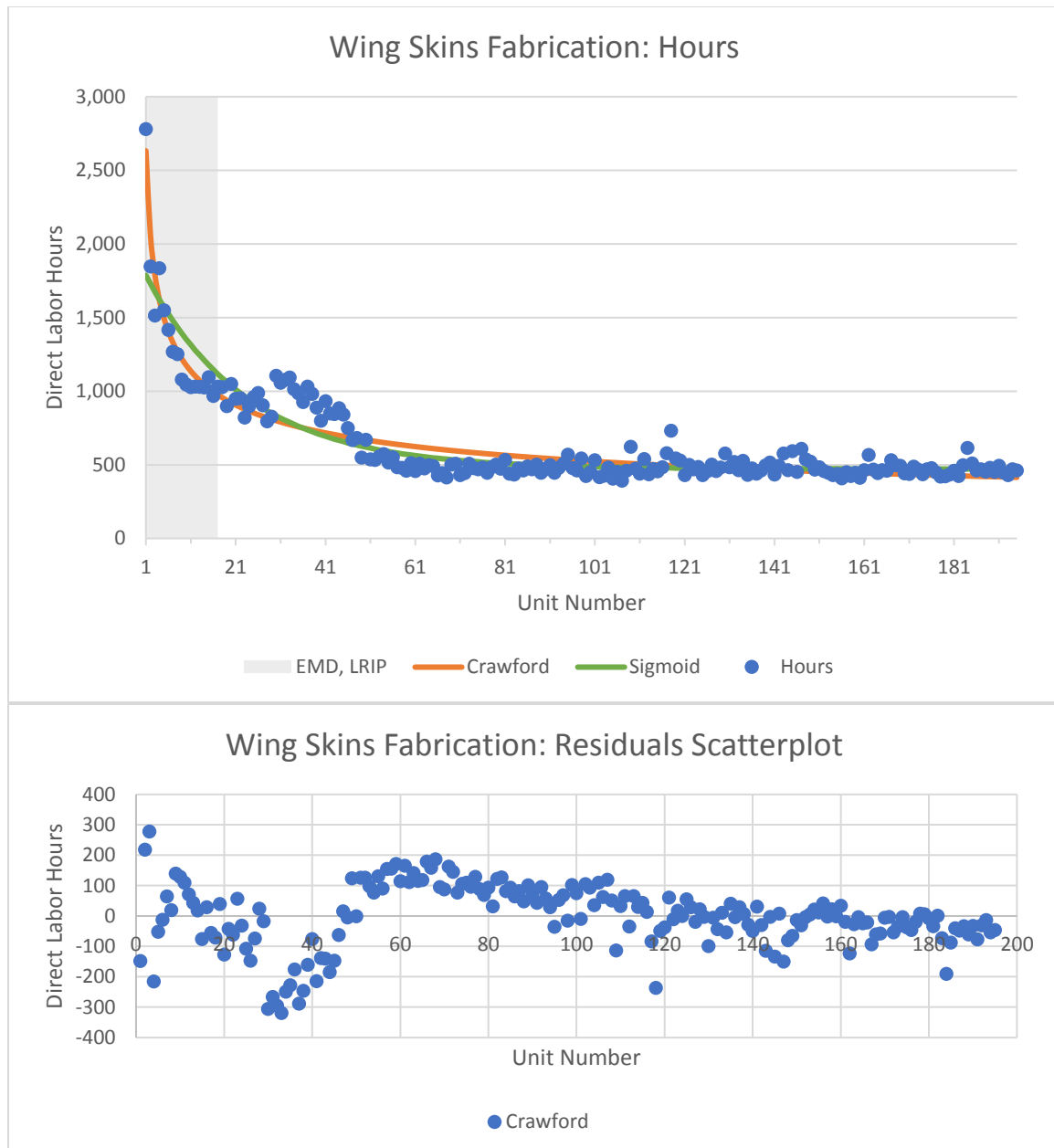


Figure 10. Wing Skins Fabrication Hours (Crawford and Sigmoid)

Figure 10 upper panel, shows that the power function is more adept than the exponential Sigmoid function for approximating both the near vertical early phase decline and the horizontal component of the data set. However, the curve does not do well by comparison during the horizontal component as the Sigmoid function. The process stabilizes nearby unit number 60 and from that unit onward the Crawford curve residuals (Figure 10 lower panel) show an approximate downward sloping line that crosses the horizontal axis at the approximate center point of this range. This pattern exemplifies the original hypothesis that early units are overestimated and later units underestimated when a power function is fit to cost data. Contrastingly, the Sigmoid function handles this phase well but cannot be manipulated to produce both a near vertical drop and a curved saturation in the manner of a power curve. The Sigmoid residuals scatter plot shown in Figure 11 displays the closer approximation of the steady state from the 60th unit onward to program completion.

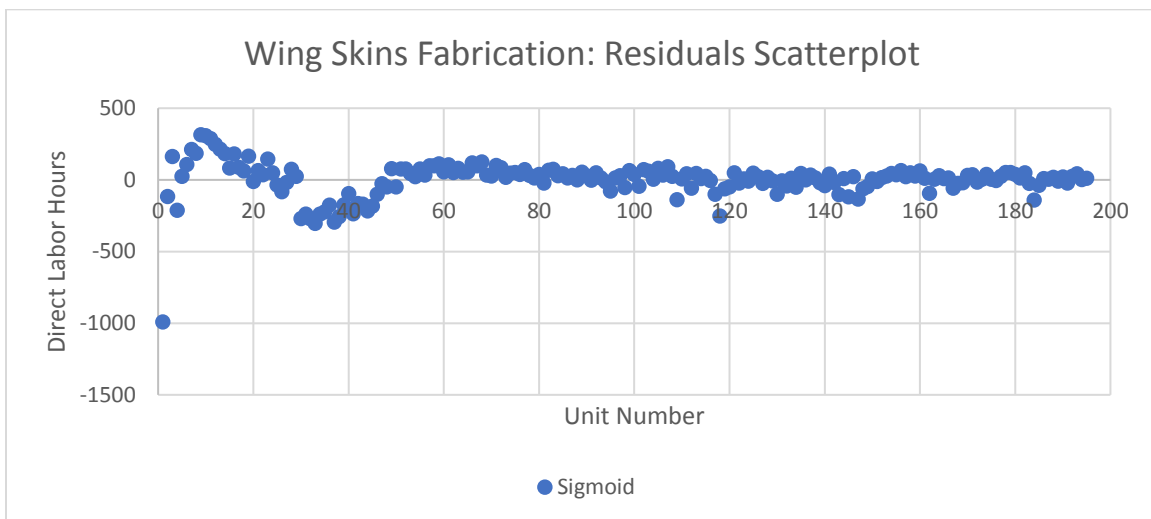


Figure 11. Wing Skins Fabrication Hours (Sigmoid Residuals)

When a production process stabilizes, it can be reasonably approximated with descriptive statistics. Using the wing skins fabrication process as an example, from the 60th unit until process termination the average cost was 477 labor hours and the standard deviation was approximately 50 hours. Figure 12 displays this relationship with a residuals scatter plot for the average (477 labor hours) starting at the 60th unit to until the last.

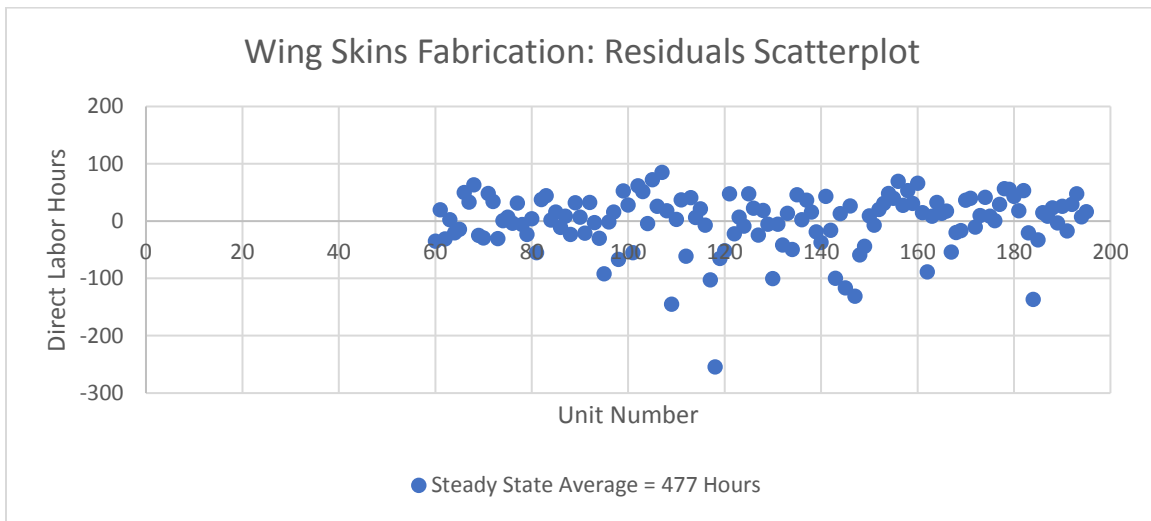


Figure 12. Wing Skins Fabrication (Steady State Residuals)

There were several outlying points (two standard deviations from the mean), and had they been absent the average unit cost would have been 468 hours with a standard deviation of approximately 32 hours. The wing skins are made of carbon fiber, and were fabricated by a programmable tape (carbon fiber) laying machine. When a replicable and satisfactory process was identified, it was maintained for the program's duration (Cantwell, 2007:12). Assuming that the process was unaltered once a steady state was achieved, then the mean and standard deviation would have been a reasonable cost estimate for this segment of the

program. The basic shape of a mathematical function and its ability to be manipulated to fit a data set is what ultimately determines its suitability or preference to an alternative.

Fitting a mathematical function to a data set helps to explain what happened but it does not tell you why it happened. For example, the steady state portion of the wing skins production process could have been well explained with its mean and standard deviation. However, if management were to fundamentally alter the process it would invalidate the relationship, and only after having collected adequate information could the new trend be substantiated. Cost analysts frequently use the production process data of one program to estimate another. Often the focus of the estimate is the attributes of the item in question. The production process itself, however, has just as much if not more influence over cost. Additionally, the learning curve (production process change) from one program may not necessarily help describe another. Production process change generally is not the result of happenstance, but deliberate management decisions. Added focus to understanding the production process can be beneficial to developing improved cost estimates.

Learning curve analysis is commonly performed, as was done in this research thus far, using the unit number or production increment number as the independent variable. A production process, however, is generally paced by time. Production rates and production cycle time often fluctuate. Unit based production process analysis could potentially result in misunderstanding process behavior depending on the time disparity between each unit. Time sequenced learning curve analysis, on the other hand, presents a production process in its most unbiased form. It shows production just as it occurred and allows an analyst to better understand the behavior of a production process. To illustrate this concept, the time

sequenced learning curve for the aft fuselage final assembly process is shown below in Figure 13.

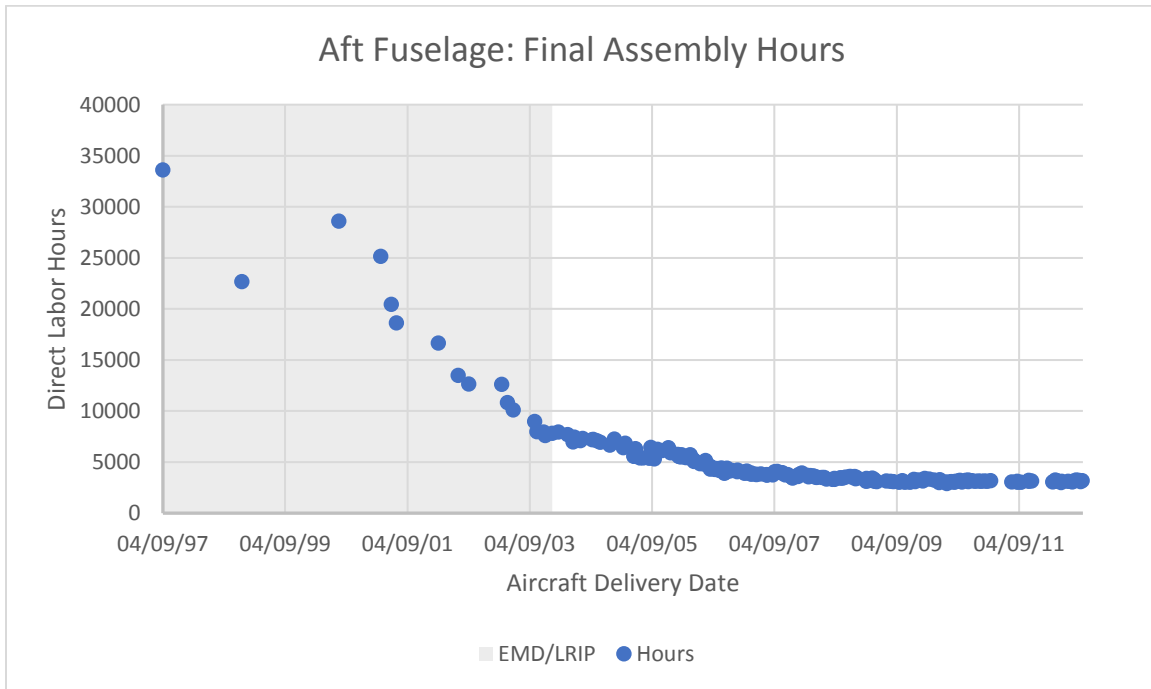


Figure 13. Aft Fuselage Final Assembly Hours - Time Sequenced

Figure 13 displays the labor hour content for each unit of production plotted along the horizontal axis by its respective aircraft final delivery date. Completion dates for each major Corporation B responsible component is unknown, but the aircraft final delivery date is a reasonable proxy. The segmented production system between Corporation A, Corporation B, and the various other parties had to be reasonably well synchronized, otherwise inventory shortages and excesses would have built throughout the system. This presentation method reveals important observations that the ordinal measured alternative (Figure 8) would not have shown. First, it clearly distinguishes the difference between the

EMD/LRIP segment and full rate production as indicated by the frequency of observations. Second, it provides a better appreciation of the relative amount of time spent in each segment. Approximately 9% of total units were produced in EMD/LRIP but it consumed 42% of total project time. Finally, it shows that the labor cost decline was not as dramatic as shown in Figure 8, but occurred over a longer period of time. An ordinal scale compressed the EMD/LRIP phase relative to the program balance, altering its meaning, and accounts for the stark difference in appearance. Time phasing also facilitates throughput analysis. Figure 14 shows trailing twelve-month production for the quarterly period indicated.

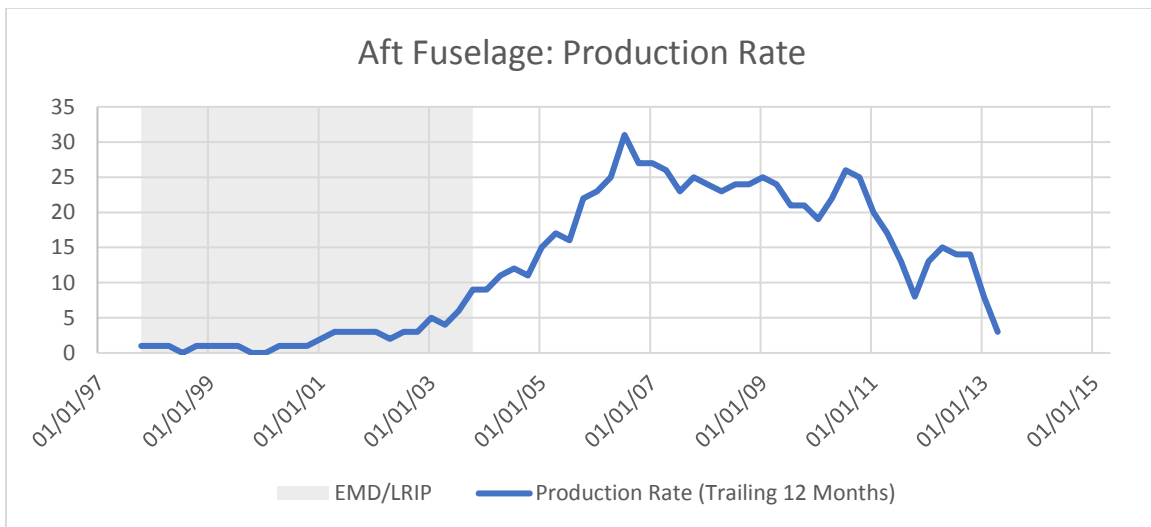


Figure 14. Aft Fuselage Production Rate (Trailing Twelve Months)

The production rate increase would imply a corresponding increase in productive capacity. Likewise, the combination of decreasing unit production costs and an increasing productive capacity (implied by the rate and price change) would support an economies of scale argument – that Corporation B transitioned from one position on their average total

cost curve to another. The “learning by doing” argument would explain that direct labor workers collectively improved their task performance resulting in an approximate 91% reduction of the labor hour content for aft fuselage final assembly from 33,614 hours for the initial unit to a mean of 3,134 for the final 65. Although this very well may have been the case, other information would suggest otherwise. An 8 August 2006 press release states the total man hour requirement to build aft fuselage units had decreased by 89% since the first delivery in October 1996. The press release attributes this reduction to the lean production principles, industrial design, and capital investment. The article goes on to attribute reductions to a late 2003 transition from massive fixed assembly jigs to smaller flexible cart tooling, time savings from electron-beam welding which reduced the need for traditional fasteners by 75%, and an automated laser-guided machine to drill holes for the remaining fasteners (Cantwell, 2006). The implications are that labor productivity improvement probably had more to do with the production process design and capital investment (trading labor hours for more advanced capital) than the collective improvement of direct laborers. Figure 15 displays a conceptual graphic summarizing the relationship between long run average total cost, productive capacity, and unit cost.

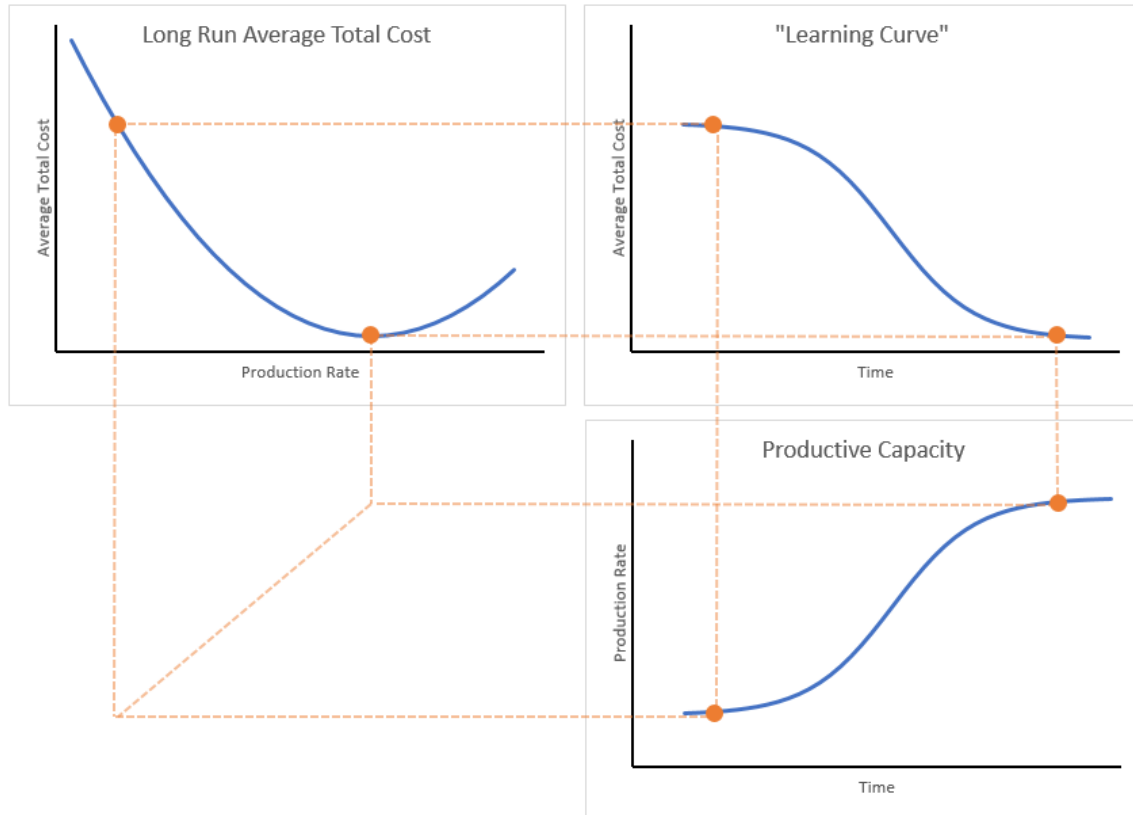


Figure 15. Relationship of Average Total Cost, Production Cost, and Production Rate

Figure 15 expresses the underlying managerial economics theory behind the ideas presented regarding Corporation B's operation. The long run average total cost curve depicts the economies of scale concept – declining long run average costs with increased throughput (Baye, 2010:185). When the ATF program began, Corporation B likely used a relatively higher labor proportion production process than what they had during full rate production. Only after selecting the full rate production process design were resources committed (capital and additional employees). With sufficient time for capital emplacement and locating the employees, the production process shifted from the initial low rate point to the higher rate production point and likewise benefitted from the labor

cost reduction that accompanied that transition. The missing component to the analysis is the number and cost of machine hours in both the early EMD/LRIP phase and later in full rate production. The change in capital operating expense and the amount of capital investment would help complete the analysis. From this perspective, learning curve analysis is not particularly distinct from a manufacturing cost estimate. Learning curve analysis as presently practiced is concerned with how the production process changes over time. Manufacturing and learning curve analysis attempt to answer four basic questions: what does the production process look like now, what will it look like in the future, how long will it take to get there, and how much resource growth (capital and labor) is required for the transition?

To better describe a time sequenced production process, curve fitting analysis can be applied. A time sequenced presentation of cost data has the benefit of depicting events as they actually occurred in real time. Figure 16 displays labor hours for aft fuselage final assembly fit with the sigmoid function.

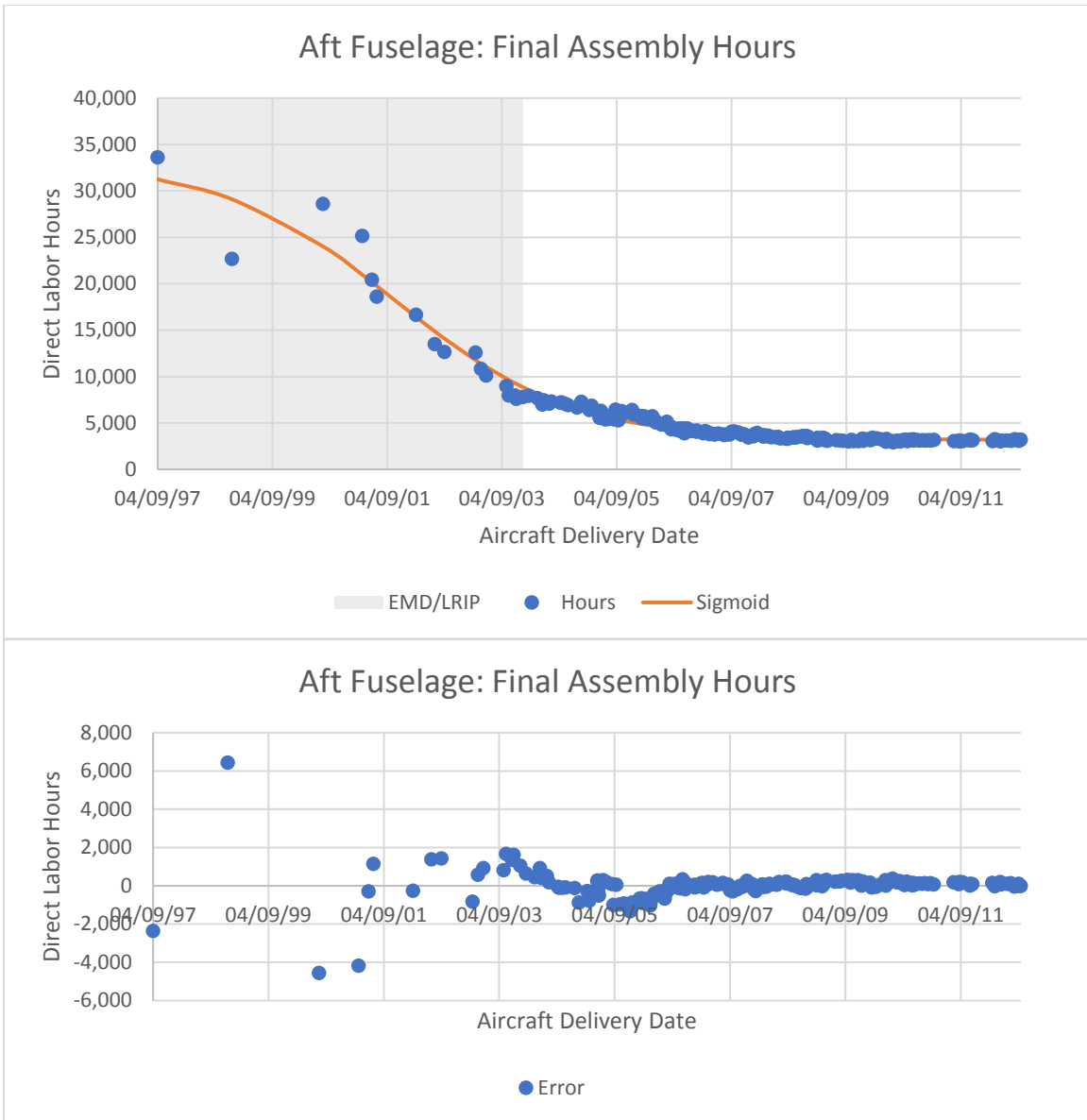


Figure 16. Aft Fuselage Final Assembly Hours - Time Sequenced (Sigmoid)

For time-sequenced production data, the sigmoid function was the preferred alternative. As before with the unit ordered analysis, this is purely a case of how well any particular mathematical function can fit data based upon its shape, and does not necessarily imply a systemic relationship to the production process. In situations which the time between cost observations is approximately equal, then unit based (ordinal) and time based sequencing

of the independent horizontal axis would constitute the same analysis. Table 5 shows the regression summary statistics and pairwise ANOVA results. Given the shape of the data, none of the power curve based variants were appropriate for time sequenced curve fitting. The pairwise ANOVA test was run for the time sequenced sigmoid regression compared to the other three combinations. For this particular data set the ordinal sequenced Carr curve is preferred to the time sequenced sigmoid function, but not to a statistically significant level. A pairwise ANOVA p-value less than an α of 0.05 indicates the time sequenced sigmoid is statistically preferred. A p-value greater than $1 - \alpha$ indicates that it is not.

Table 5. Aft Fuselage Assembly - Time Sequenced Regression Summary Statistics

	Ordinal Sequenced		Time Sequenced	
	Carr	Sigmoid	Carr	Sigmoid
R^2	0.9663	0.9317	0.8906	0.9629
σ_ε	765	1,089	1,378	803
Paired ANOVA	0.7483	0.0000	0.0000	-

Summary and Conclusion

Learning curve analysis is used to describe and estimate serial production process performance. This analysis is commonly performed using bivariate regression analysis. The vertical dependent axis is the cost measure and the horizontal independent axis is the ordinal number of unit production. Production number and the cost measure are plotted in two-dimensional space. Regression analysis is then performed to fit chosen mathematical functions to the data set. The fit quality or ability for any particular function to fit a data set depends primary on that functions basic shape and ability to conform to the shape of

the data set. A fitted function can then be used to describe the data and make extrapolated inferences or estimates about the near future behavior of the underlying process. The fitted function, however, does not infer a systemic relationship to the underlying process because the function is not sensitive to the central factors that control the behavior of a production process such as labor and capital levels. Ordinal sequenced learning curve analysis could potentially alter or distort the understanding of the underlying production process. This is most prevalent in situations where there is a great amount of time disparity between unit production rates. Time sequencing can remedy this problem and provide an analyst with a true picture of the production process behavior. Similarly, regression analysis can be used to fit a function to the data to provide further understanding and extrapolative estimating. Time sequenced data can also provide information on the production rate over time and allows for easier comparisons to a production schedule. Although a time sequenced data analysis can provide additional fidelity above ordinal sequencing, it still suffers the same shortcoming in that certain data elements necessary to complete the analysis are missing – specifically, the level of resources (labor and capital) present in the production process. The added information of resource levels would aid an analyst in developing a complete picture of what is occurring in a production process. Implicitly the goal is understanding the prevalent production function in a firm at any given time and using that information to understand and estimate cost.

V. Findings and Conclusions

Introduction

In the context of a serial production process the term learning curve is in reference to the observed decline of individual unit cost with incremental production. This effect is most commonly attributed to increased knowledge and experience producing the product and its chosen production method. Learning curves are of particular interest to managers, and learning curve data is often modeled mathematically to create extrapolative estimates for an existing ongoing program or to build an analogous estimate for a planned program. The initial motivational for this research was to determine if one particular mathematical model among those most commonly used for this task was more consistently preferred to others for modeling learning curve data. However, as the research progressed it became apparent that how well any particular mathematical function could fit to the cost data of a production process was not nearly as important as identifying underlying systemic drivers for the production process in question. Although a well fit mathematical function can be useful for describing data and creating extrapolative estimates, this utility depends on the underlying systemic factors that drive the observed cost behavior to remain constant. Any deviation in effective behavior of the most relevant systemic factors would invalidate the relationship. Additionally, using a mathematical function that was well fit to the learning curve data of one particular production process to build an analogous estimate would be

limited without relevant quantitative data regarding the production process drivers. The cost to produce an item is just as much a function of production process as the item itself.

Learning Curve Theory and Analysis

General learning curve theory and analysis was initially developed in the realm of human psychology. Researchers were developing techniques to measure and understand human learning, and how learning affected memory and sensory-motor task performance. Early learning performance studies were focused on individual performance but the basic principles have since been applied to entire organizations such as an industrial production process. Performance is typically modeled as a dependent variable of time. The implicit assumption is that learning is accumulated and actively applied to task performance over time, and that the variable of interest is the individual or the organization being observed. This relationship reasonably allows an analyst to model performance with a mathematical function with a single independent variable. However, if the particulars of task execution changed during the observation period, then performance is not just a function of learning but also that of the nature and timing of the change in task execution. In the context of a production process, if the design of the process itself changed in terms of how production was carried out, then the impact of those changes as well as worker proficiency influence the performance of that process. The more commonly used learning curve models used to analyze production processes do not consider the performance effects of alterations made thereto. Including quantitative input regarding the relative productive capabilities of the production process into the model perhaps could improve the analysis and modeling.

Advanced Tactical Fighter (ATF) Analysis Results

The primary research objective is to performance compare and create a preference ordering for the most commonly used mathematical learning curve models for developing probabilistic models of serial production process data. The goal is improved appreciation of the systemic cost drivers of a production process, their relationship to cost, and present modeling methods. To that end, four of the more common variants of the power function and the sigmoid function, an exponential function, were fit to the data of five production activities of the Air Force Advanced Tactical Fighter (ATF) program. No definitive trend in terms of universal favorability of one curve over another was identified. Generally, the preference of one function or another was situational dependent and influenced by certain dynamics of the analytical process. The most influential factors are the visual form of the data sets when plotted, the visual form of the function to be fit and how well it can adhere to the data, and the sequencing choice of the horizontal axis – either with unit completion ordering or the time of completion. Unit of completion sequencing of the horizontal axis can alter the appearance of the learning curve, particularly if significant amounts of time dispersion exists between data points. In the five production activities analyzed, unit costs decreased from program initiation to its maturity, paralleling the transition from low rate early phase production to high rate production. Four of the five activities stabilized as the program matured. Early phase production rates were lower than full rate production, and ordinal data sequencing visually distorted the rate of unit cost decline. When the data sets are ordinally sequenced, generally the power curve models were preferred to the Sigmoid function. The opposite is true when the data is sequenced by the time of unit completion.

In both cases the desirability of one function or another is totally a function of the data's visual appearance and the function of choice's ability to conform to it. Time sequencing, however, presents the production process as it actually occurred and allows for the visual assessment of production rate, production breaks, and parallel unit production. Lastly, the curve fitting process calculates the function parameters which minimize the total variance between the fit function and data set. In most cases the calculated constants do not adhere to the constraints set within the scope of each model, and frequently produced values that could not be reasonably explained particularly in the case of the power curve models.

Analysis Conclusions

How well a mathematical function can be fit to a data with the regression process depends on the visual shape of the data with respect to its sequencing along the horizontal axis, and the basic shape and adaptability of the math function that is being targeted to fit that data. Fit quality does not translate to a systemic relationship with the underlying cost drivers of the production process. The primary drivers of a production process are relative resource levels (labor and capital), the process arrangement and design, and learning both of the direct labor force as well as the management and support engineers. Curve fitting is useful for describing and predictive extrapolation but it does not necessarily help develop a broad systemic understating across multiple production processes. A fit curve describes the behavior of the process but does not necessarily correspond to the underlying process drivers which to an extent limits its predictive use. A fit curve would not anticipate major changes to the production process. Typically, management alters the production process,

which ultimately alters the process results but these changes do not feed to a fitted model. Specifically, the transitioning of a production process from an initial lower rate to its final planned full rate arrangement, which entails the addition and significant rearrangement of resources (labor and capital), accounts for a significant portion of production cost decline commonly observed in learning curve analysis. Production process analysis would likely improve with more focus on understanding on the production process. Predictive learning curve analysis commonly entails identifying production process behavior over time using the curves of past similar efforts. The comparative focus is the production item, but often little attention is given to the attributes of the production system itself. Understanding the production process in its present state, the process as it will be implemented in the future, and the amount of time it will take to transition is ultimately what a cost analyst is trying to understand.

Future Research Possibilities

The research findings and conclusions indicate that additional study in the area of identifying metrics and relationships that could assist in understanding and explaining the drivers of a production process would be useful. Gathering additional information related to the productive ability of a production process such as labor (number of employees) and capital (total investment or capital operating cost) would be useful for increased insight to the relationship between cost and productive capability. This information could possibly be used to develop a parametric relationship between unit costs at various locations in the production time line and the corresponding resource levels. The reliable identification of

this relationship could enhance production process analysis and assist with developing improved cost estimates.

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14. ABSTRACT Learning curves are used to describe and estimate the cost performance of a serial production process. There are numerous different models and methods, however, it is not definitively known which is preferred. The research objective is to performance compare the more common learning curve models. The research goal is improved understanding of the systemic cost drivers of a production process, their relationship to cost, and present modeling methods. The research method is qualitative analysis combined with statistical regression modeling. The research identified that preference for one function or another depended upon the shape of the data and how well a model formulation could be made to fit that shape. This depended upon the model's basic shape and the available parameters to alter its appearance. The typical learning curve model assumes that cost is a function of time but commonly omits factors such as production process resources changes (capital and labor) and its effect on cost. A learning curve model that includes the effects of resource changes would likely provide higher estimative utility given that it establishes a systemic relationship to the underlying production process. Additional research and data is required to further develop this understanding.		
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